SOLUTIONS

TOOL 1: Gothic Art

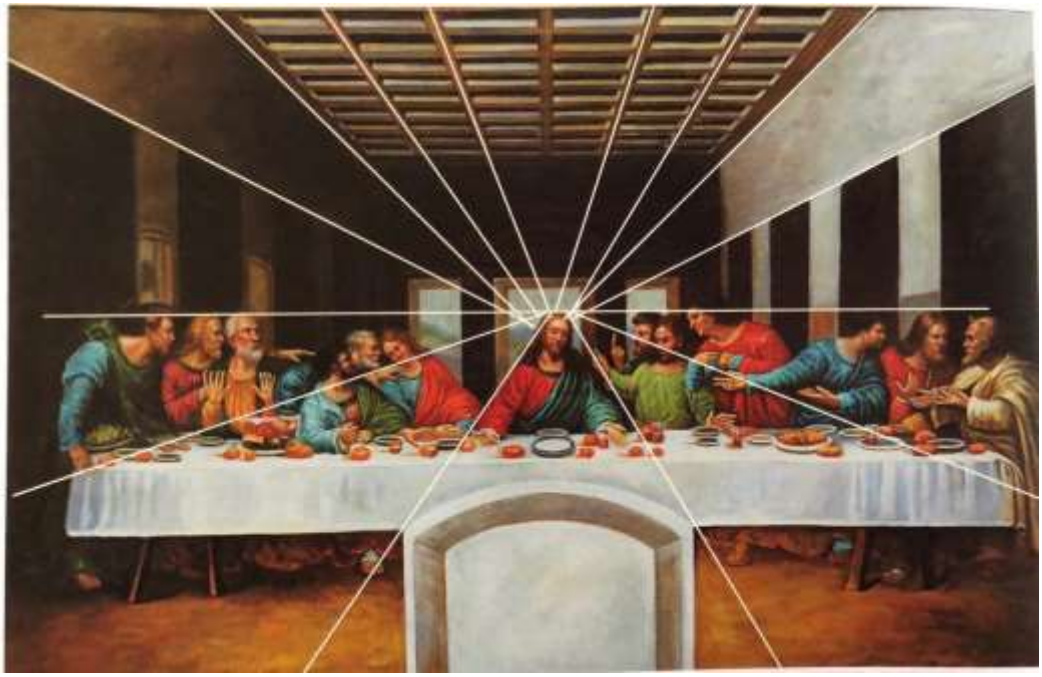
[explanation given within the tool]

TOOL 2: Islamic Art and Geometry

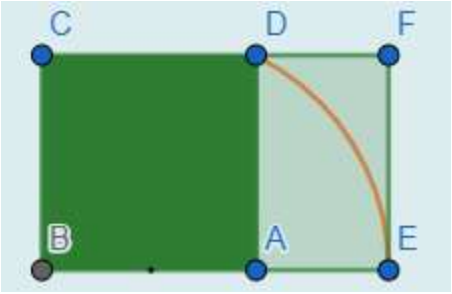
[explanation given within the tool]

TOOL 3: Renaissance Art and Geometry

A)



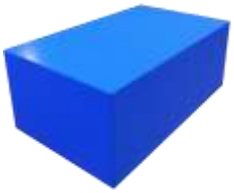
B)



- $\frac{2,7}{b} = 1,618$
- 2,7 = 1,618 * b
- $b = \frac{2,7}{1,618}$
- b = 1,669

C)

This is a cone, as it is curved, it is not a polyhedron!



Yes, this is a rectangular prism

$F + V - E = 2 \Leftrightarrow 6 + 8 - 12 = 2$

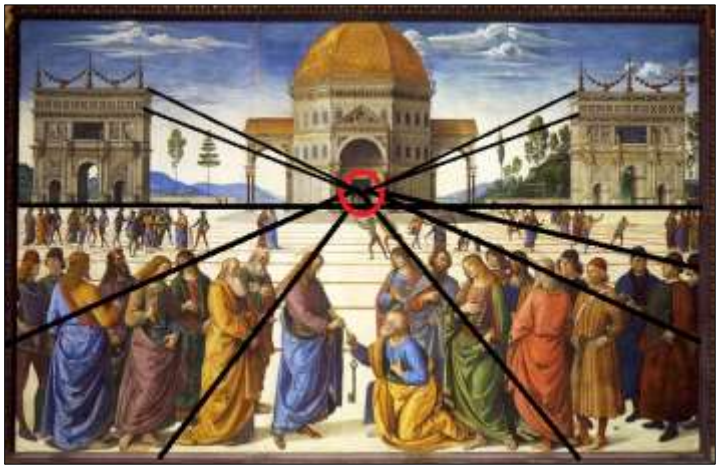
Yes, this is a triangular prism

$5 + 6 - 9 = 2$

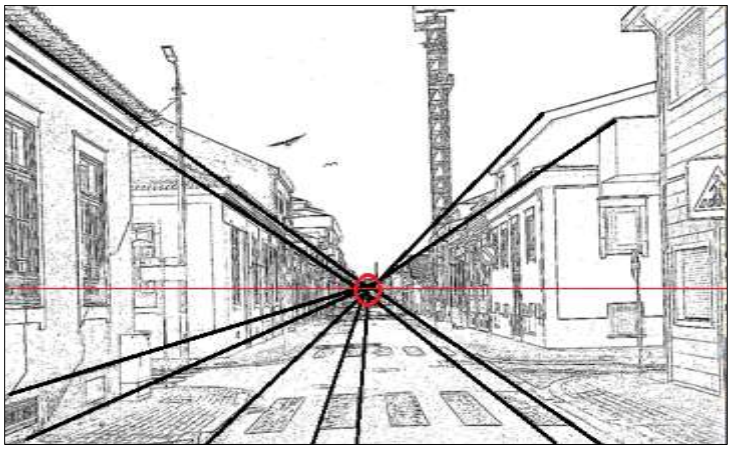
TOOL 4: Polyhedra and Perspective

TASK 1

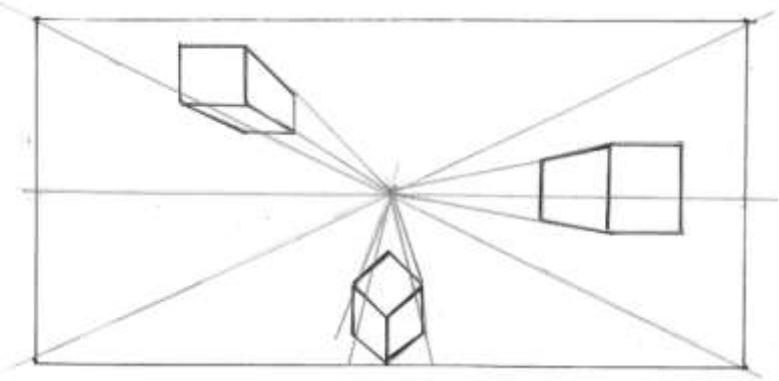
1.1 [Possible answer]



1.2 [Possible answer]

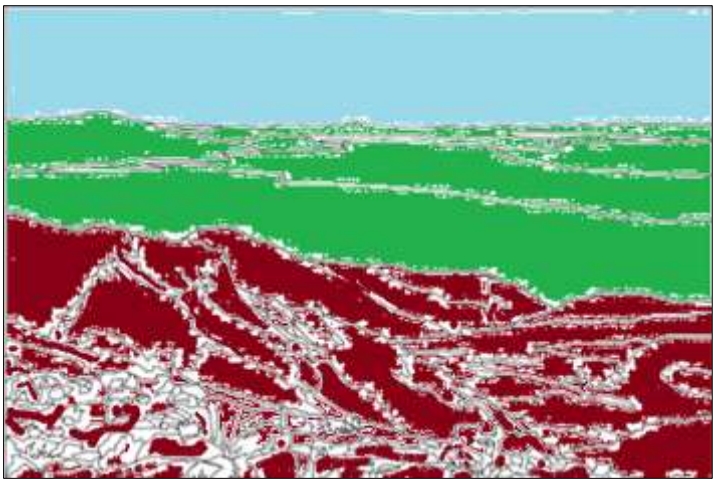


1.3 [Possible answer]



TASK 2

[Possible answer]



TASK 3

Fig. 20 – No perspective;

Fig. 21 – Aerial perspective;

Fig. 22 – Linear perspective.

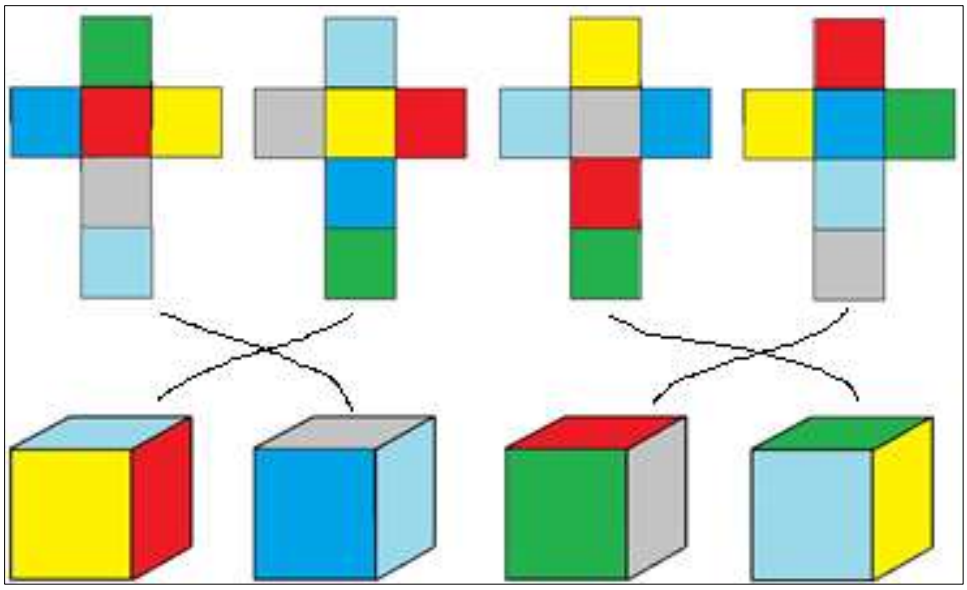
TASK 4

Platonic solid	Number of faces (F)	Number of vertices (V)	Number of edges (E)	E + 2	F + V
Hexahedron	**6**	**8**	**12**	**14**	**14**
Tetrahedron	4	4	6	8	8
Octahedron	8	6	12	14	14

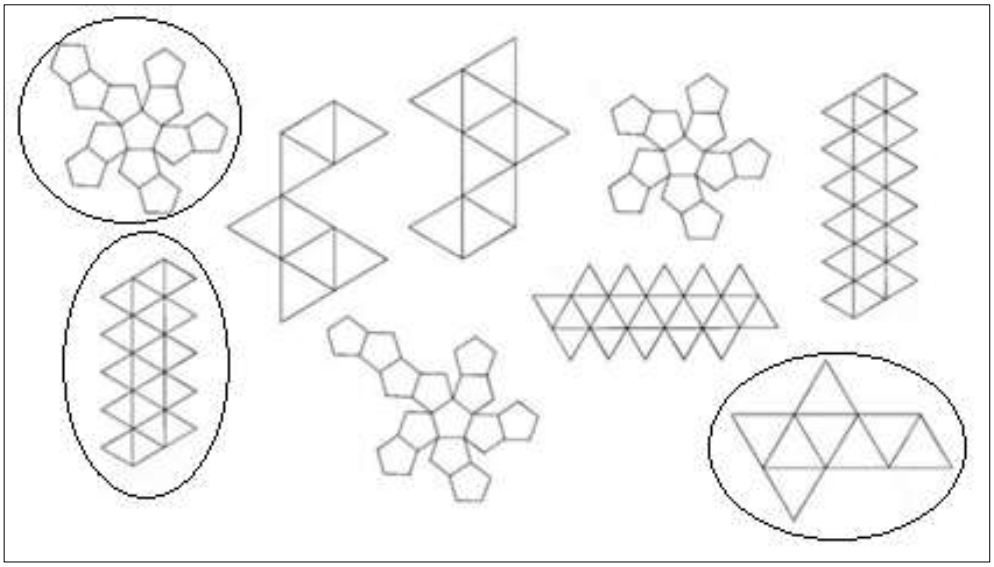
Dodecahedron	12	20	30	32	32
Icosahedron	20	12	30	32	32

TASK 5

5.1



5.2



TOOL 5: Origami and Spatial Relations

Thales Theorem

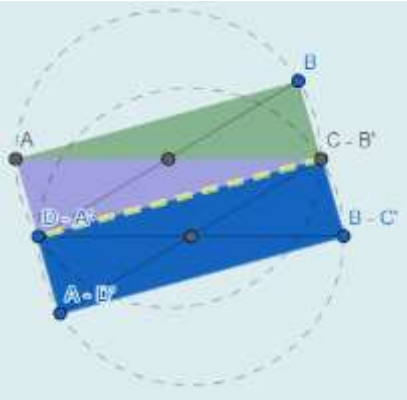
a) Yes

b) Yes

c) That the angles ∠ABC and ∠ADC are right angles!

d) [Possible answer]

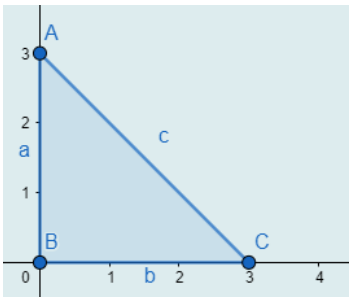
If we fold the paper on segment DC:



We can project the same rectangle below, which will be called A'B'C'D' in which:

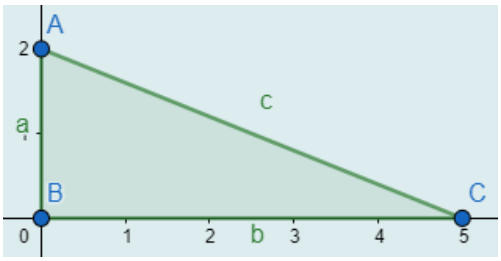
- Segment AB will be folded onto segment D'C'
- Segment DC will become the segment A'B'

Pythagoras Theorem



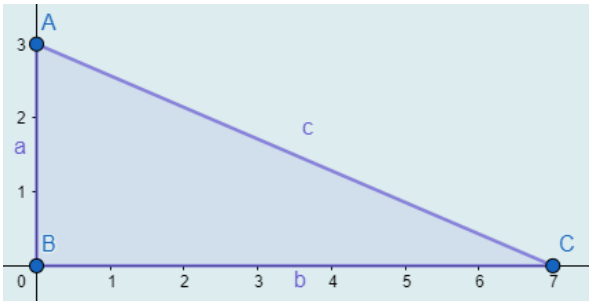
$3^2 + 3^2 = 9 + 9 = 18$

$c = \sqrt{18} = 4.2426$



$2^2 + 5^2 = 4 + 25 = 29$

$c = \sqrt{29} = 5.3852$



$3^2 + 7^2 = 9 + 49 = 58$

$c = \sqrt{58} = 7.6158$

TASK

a) Yes

b) Yes

c) Yes

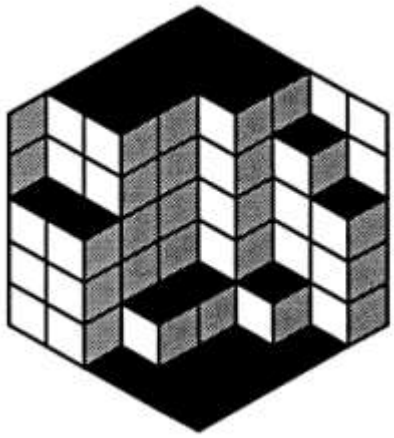
d) Yes

TOOL 6: The Mathematical Art of M. C. Escher

TASK

1) The number of sweets is equal.

2)



3) The observation is that it is transformed into a three-dimensional shape. Cube. Calissons tend to behave like faces of a unit cube (cubes of which the edges are of length one).

TOOL 7: A Synchronous Exhilaration of Art Math Masterpiece

TASK

A → 6

B → 7

C → 3

D → 4

E → 1

F → 2

G → 5

TOOL 8: Golden Ration in Arts and Architecture

TASK 1

a) We start from **φ=** $\frac{a}{b} = \frac{a+b}{a}$. This equation could be written as $\frac{a}{b} = \frac{a}{a} + \frac{b}{a}$

But it's already known that $\frac{a}{b} = \varphi$, whereas $\frac{b}{a} = \frac{1}{\varphi}$

So, we get $\varphi = 1 + \frac{1}{\varphi}$

b) We start from: $\varphi = 1 + \frac{1}{\varphi}$

⇨ $\varphi^2 = \varphi + 1$
⇨ $\varphi^2 - \varphi - 1 = 0$

We use the formula to identify the two roots (solutions):

$\varphi 1,2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, whereas a=1, b=-1, c=-1

⇨ $\varphi 1,2 = \frac{1 \pm \sqrt{1^2 + 4}}{2}$

We keep only the positive solution (length)

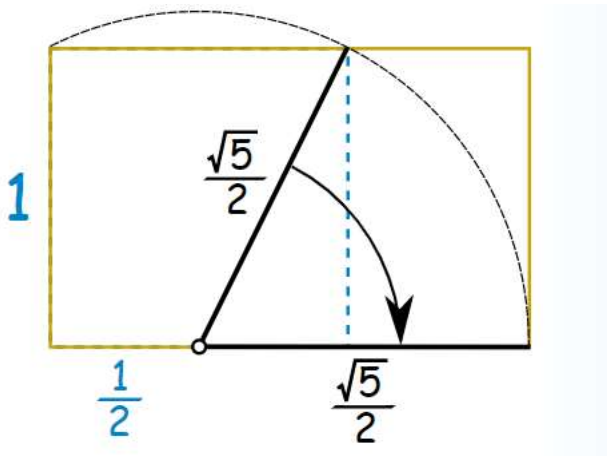
$\varphi = \frac{1+\sqrt{5}}{2}$=1.618

TASK 2

a) to g)

e) Use Pythagoras theorem to find the length of the line:

$$\sqrt{(1^2 + (\frac{1}{2})\text{^}2)} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$$



h) Remember that $\varphi = \frac{a+b}{a}$, whereas: $a + b = \frac{1}{2} + \frac{\sqrt{5}}{2}$ and a=1

$\Rightarrow \varphi = \frac{1+\sqrt{5}}{2} = 1.618$

TOOL 9: Art Imaging Through the Use of Functions

TASK 1

[Possible answer]

C	(x+4)^{2}+(y-1)^{2}=1^{2} {x<-3.25}	
L	x=-2.5{0<y<2}	y=0 {-2.5<x<-1.5 }
A	2x+2\ {-1<x<0 }	-2x+2\ {0<x<1 }
	y=1 {-0.5<x<0.5 }	
S	(x-2)^{2}+(y-1.5)^{2}=0.25 {y>1.5 }	(x-2)^{2}+(y-1.5)^{2}=.25 {1.5 < x<2 }
	(x-2)^{2}+(y-0.5)^{2}=.25 {2 <x<2.5 }	(x-2)^{2}+(y-0.5)^{2}=.25 {y<0.5 }
S	(x-3.5)^{2}+(y-1.5)^{2}=0.25 { y >1.5 }	(x-3.5)^{2}+(y-1.5)^{2}=.25 {3 < x < 3.5 }
	(x-3.5)^{2}+(y-0.5)^{2}=.25 {3.5 <x<4 }	(x-3.5)^{2}+(y-.5)^{2}=.25 {y<0.5 }

M	x=-6{0<y<2}	-1x-4{-6 <x <-5}
	x+6{-5 < x < -4}	x=-4{0<y<2}
A	2x+6{-3 < x <-2}	-2x+-2{-2<x<-1}
	y=1{-2.5<x<-1.5}	
T	y=2{0<x<2}	x=1{0<y<2}
H	x=3{0<y<2}	x=3 {0<y<2}
	x=3{0<y<2}	
S	(x-6)^{2}+(y-1.5)^{2}=0.25{y>1.5}	(x-6)^{2}+(y-1.5)^{2}=.25{ 5 < x < 6}
	(x-6)^{2}+(y-0.5)^{2}=.25{ 6 < x< 6.5	(x-6)^{2}+(y-0.5)^{2}=.25{y<0.5

TASK 2

[Open ended question]

TOOL 10: Patters in the Portuguese Pavement

TASK 1

1.1 Rotation of 180º (half turn).

1.2 Vectors $\vec{u}$ and $\vec{t}$.

TASK 2

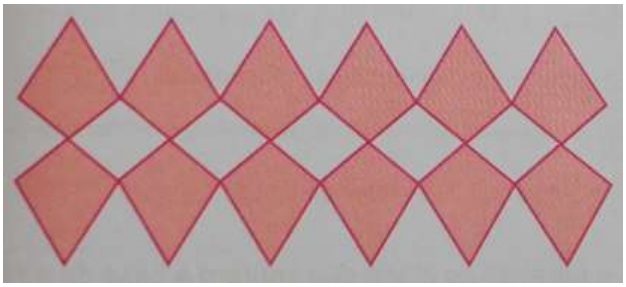
2.1 Glide reflection and translation.

2.2 Vertical reflections, horizontal reflections, rotation of 180º (half turn) and translation.

2.3 Vertical reflection, glide reflection, rotation of 180º (half turn) and translation.

TASK 3

[Possible answer]



TASK 4

I: Translation;

II: Vertical reflection and translation;

III: Horizontal reflection and translation;

IV: Rotation of 180º (half turn) and translation.

TOOL 11: Fractals and Dimensions

Archimedes used the fact that the circumference of a circle is bounded by the perimeter of an inscribed polygon and the perimeter of a circumscribed polygon. This fact was used to approximate π.

Archimedes used a 96-sided polygon to find following approximation:

$$\frac{223}{71} < \pi > \frac{22}{7}$$

It is easy to use this method by using trigonometry, Archimedes however only used geometry and Greek numerals.

TOOL 12: Fibonacci Spiral in Visual Arts

TASK 1

M(7)= 13 pairs of rabbits

M(12)=144 pairs of rabbits

M(25)=75025 (since M(22)=17711 and M(24)=46368)

TASK 2

a) 2 ways (1-1-1, 1-1-2)

b) 3 ways (1-1-1-1, 1-2-1, 1-1-2)

c) 55 ways

TOOL 13: Paper Folding Geometry

Body	Number of sides	Shape on the side surfaces	Number of degrees in the side surface corner	The angle sum in the corners of the body
tetrahedral	4	equilateral triangle	60	3*60=180
hexahedron	6	square	90	360
octahedron	8	equilateral triangle	60	180
dodecahedron	12	pentagon	108	540
icosahedron	20	equilateral triangle	60	180

TOOL 14: Ratios of Frequencies of Musical Notes

It' s been proven that $r_3 = r_2.r_1$

- ⇨ $3/2= 6/5. r_1$
- ⇨ $r_1 = 5/4$

But we know that $r_1=f_1/f_0$ and that $f_1=5$ Hz

- ⇨ $f_0=4$ Hz

TOOL 15: Pythagorean Tuning and Ratios

TASK

1)

C	D	E	F	G	A	B	C'	D'	E'	F'	
8:	9										x2
	2:				3						x9
16:					27						

2)

C	D	E	F	G	A	B	C'	D'	E'	F'	
16:					27						x2
					2:				3		x27
32:									81		

3)

C	D	E	F	G	A	B	C'	D'	E'	F'	
32:									81		x2
		1:							2		x81
64:		81									

4)

C	D	E	F	G	A	B	C'	D'	E'	F'	
64:		81									x2
		2:				3					x81
128:						243					

5)

C	D	E	F	G	A	B	C'	D'	E'	F'	
128:						243					x2
						2:				3	x243
256:										729	

6)

C	D	E	F	G	A	B	C'	D'	E'	F'	
256:										729	x2
			1:							2	x729
512:			729								

You can now give the ratios for all the notes!

C	D	E	F	G	A	B	C'
1/1	9/8	81/64	729/512	3/2	27/16	243/128	2/1

TOOL 16: Numerical Series in Harmonic Series

TASK 1

Follow the link for an example:
https://www.youtube.com/watch?v=AQJw95-H9mM

TASK 2

2.1) $u_n = 5n - 2$

2.2) $u_n = -3n + 1$

2.3) $u_n = \frac{1}{n}$

2.4) $u_n = \frac{1}{3n}$

2.5) $u_n = \frac{1}{n^3}$

TASK 3

3.1) $3, 6, 9, 12, 15, \ldots$

3.2) $4, 9, 14, 19, 24, \ldots$

3.3) $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots$

3.4) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots$

TOOL 17: Music and the Golden Ratio

TASK

1) $\varphi = \frac{(a+b)}{a}$

$a = \frac{(a+b)}{\varphi}$

$a = \frac{55}{\varphi} = 34$

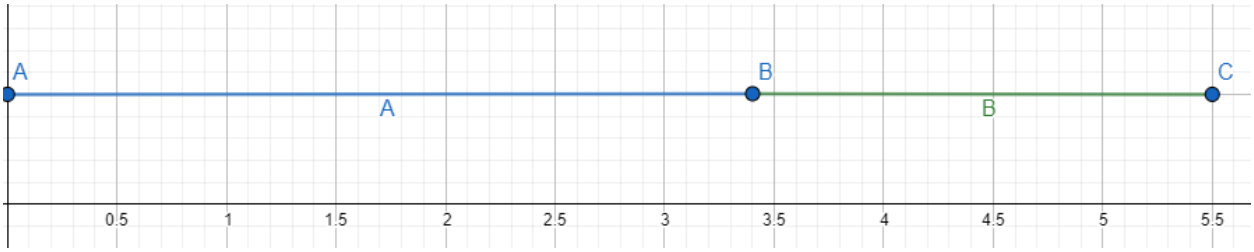
2) $\varphi = \frac{a}{b}$

$b = \frac{a}{\varphi}$

$b = \frac{34}{\varphi} = 21$

3) $\varphi = \frac{55}{34} = \frac{34}{21} \approx 1{,}618$

4)



TOOL 18: Powers in the Tempered Scale

TASK 1

1.1. -8 **1.2.** 1 **1.3.** $\frac{4}{25}$ **1.4.** 0 **1.5.** $\frac{1}{16}$

TASK 2

2.11. $4^3 \times 4^5 = 4^8$;

2.12. $(-3)^3 \times (-3)^5 = (-3)^8$;

2.13. $5^7 : 5^5 = 5^2$;

2.14. $\left(-\frac{3}{2}\right)^8 : \left(-\frac{3}{2}\right)^5 = \left(-\frac{3}{2}\right)^3$;

2.15. $(3^5)^2 = 3^{10}$;

2.16. $(3^5)^3 = 3^{15}$;

2.17. $\left(\frac{27}{8}\right)^2 = \left(\frac{3}{2}\right)^6$

TASK 3

3.1. $\left(\frac{1}{2}\right)^3$;

3.2. $\left(\frac{5}{3}\right)^{12}$;

3.3. $\left(\frac{1}{5}\right)^{11}$;

3.4. $\left(\frac{3}{5}\right)^{12}$.

TASK 4

4.1. 64;

4.2. 18;

4.3. $\frac{9}{25}$.

TOOL 19: Ratios of Frequencies of Musical Notes

a) ma+nb+sc = m log(2) + n log (3/2) + s log (5/4)

But we must consider the following properties of logarithms:

x log y = log (y)x
and log(x) + log (y) = log (xy)

So ma+nb+sc = log(2)m + log (3/2)n + log (5/4)s

ma+nb+sc = log [(2)m (3/2)n (5/4)s]

Consequently, the ratio of the random interval ma+nb+sc is (2)m (3/2)n (5/4)s

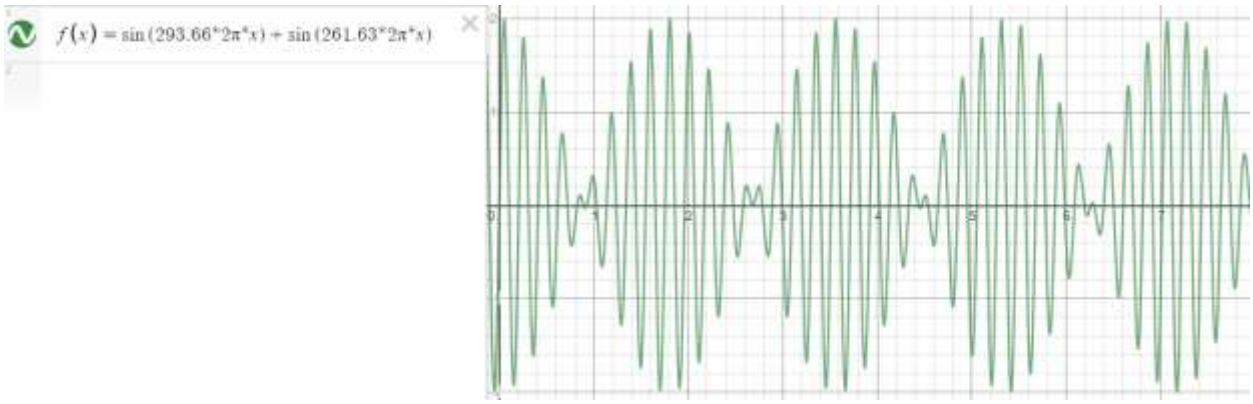
In this way, we have proven that any interval can be determined as functions of a,b and c.

b) The interval is defined as b+c, (we have taken the random interval ma + nb + sc considering that m=0, n=1 and s=1). Accordingly, the ratio is (3/2)(5/4)= (15/8)

TOOL 20: The Beat Equation

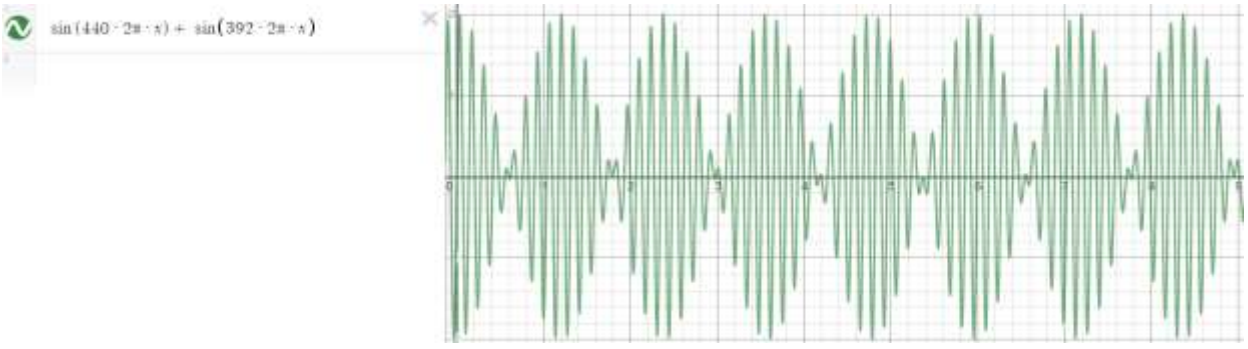
1) D and C

$$sin(293.66 * 2\pi * x) + sin(261.63 * 2\pi * x) = 2sin(277.64x * 2\pi) + cos(32x * 2\pi)$$



2) A and G

$$sin(440 * 2\pi * x) + sin(392 * 2\pi * x) = 2sin(416x * 2\pi) + cos(48x * 2\pi)$$



TOOL 21: Trigonometric Functions in Harmonic Series

TASK 1

1.1 $P = \pi$ **1.2** $P = 6$

1.3 $P = \frac{2\pi}{5}$ **1.4** $P = 2$

1.5 $P = \frac{\pi}{2}$ **1.6** $P = 2$

TASK 2

2.1. $x = \frac{5\pi}{4} + 2k\pi \; V \; x = -\frac{\pi}{4} + 2k\pi, k \in Z;$

2.2. $x = \frac{4\pi}{3} + 2k\pi \; V \; x = -\frac{\pi}{3} + 2k\pi, k \in Z;$

2.3. Impossible because $-1 \leq sin\, x \leq 1$ and $2 \notin [-1; 1]$;

2.4. $x = \frac{\pi}{12} + k\pi \; V \; x = \frac{5\pi}{12} + k\pi, k \in Z.$

TASK 3

3.1. $x = \frac{3\pi}{4} + 2k\pi \; V \; x = -\frac{3\pi}{4} + 2k\pi, k \in Z;$

3.2. $x = \frac{2\pi}{3} + 2k\pi \; V \; x = -\frac{2\pi}{3} + 2k\pi, k \in Z;$

3.3. $x = \pi \; V \; x = 3\pi.$

TASK 4

4.1. $x = \frac{5\pi}{3} + 2k\pi, k \in Z;$

4.2. $x = \frac{\pi}{8} \; V \; x = \frac{5\pi}{8} \; V \; x = \frac{9\pi}{8} \; V \; x = \frac{13\pi}{8}.$

TOOL 22: Music and Fibonacci

$$x_{16} = \frac{\varphi^{16} - (1-\varphi)^{16}}{\sqrt{5}} = 987$$

$$x_{17} = \frac{\varphi^{17} - (1-\varphi)^{17}}{\sqrt{5}} = 1597$$

$$x_{18} = \frac{\varphi^{18} - (1-\varphi)^{18}}{\sqrt{5}} = 2584$$

$$x_{19} = \frac{\varphi^{19} - (1-\varphi)^{19}}{\sqrt{5}} = 4181$$

$$x_{20} = \frac{\varphi^{20} - (1-\varphi)^{20}}{\sqrt{5}} = 6765$$

TOOL 23: Pythagoras and his Mathematical Music

TASK

6) The wavelength of the tone is 4 times the distance from the mouth to water surface, a 1/4-part wave is thereby formed in the bottle! The higher the tones, the smaller the wavelengths.

$$\frac{340\ m/s}{frequency} = \text{wavelength}$$

TOOL 24: Pythagoras and his Mathematical Music

TASK 1

6) The wavelength of the tone is 4 times the distance from the mouth to water surface, a 1/4-part wave is thereby formed in the bottle! The higher the tones, the smaller the wavelengths.

$$\frac{340\ m/s}{frequency} = \text{wavelength}$$

TASK 2

If you blow into the bottle, it will be the air vibrating. If you strike the bottle it will be the bottle that vibrates. The sound when you blow will be higher than the sound from striking.

TASK 3

Use the information (the formula) in the answer for TASK 1 and measure your bottles.

TOOL 25: Bach and the Musical Moebius Strip

TASK

a) $4\frac{2}{3}\ au$

b) $2\frac{2}{3}\ au$

TOOL 26: Bach and the Musical Moebius Strip

TASK

a) 3 turns

b) 102 cm

TOOL 27: Logarithms in the Tempered Scale

TASK 1

1.1. 6 **1.2.** 1 **1.3.** -4 **1.4.** 0 **1.5.** $-\frac{1}{2}$

1.6. 6 **1.7.** 3 **1.8.** -2 **1.9.** $\frac{1}{2}$ **1.10.** $\frac{4}{3}$

1.11. $-\frac{1}{2}$ **1.12.** -2 **1.13.** -7

TASK 2

2.11. 4;

2.12. -2;

2.13. -7;

2.14. $-\frac{19}{5}$;

2.15. -1;

2.16. $-\frac{5}{6}$.

TASK 3

3.1. 10; **3.2.** 1; **3.3.** 40.

TASK 4

-2.

TOOL 28: Basic Arithmetic in "Beasts of Burden" (The Man Who Counted, Chapter III)

TASK 1

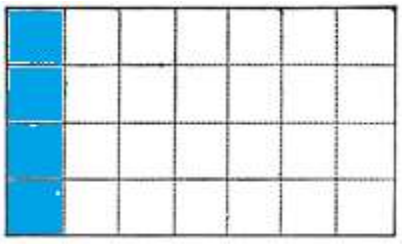
1) b

2) a

3) b

4.1) [Possible answer]

4.2) [Possible answer]



5.1) b

5.2) c

5.3) b

5.4) b

5.5) c

5.6) c

6.1) b

6.2) c

6.3) a

6.4) a

7) a

8) b

9) b

10) c

11) b

TASK 2

[Role-play]

TOOL 29: Geometry Through Euclides Elements

TASK

[Role-play]

TOOL 30: Volumes in “Seventh Heaven” (The Man Who Counted, Chapter VIII)

TASK 1

[Role-play]

TASK 2

1.1. $216\ cm^3$.

1.2. $24\ cm^3$.

1.3. $192\ cm^3$.

TASK 3

$564\pi\ cm^3$.

TOOL 31: Approaching Math Logic Through "The Lesson" of E. ionesco

TASK 1

(I)

(i) No

(ii) Yes

(iii) No

(II)

P	P'
A	A' (False-F)
Y	Y' (True- T)

(III)

P1	P2	P1 and P2
A	Y	False (F)
Y	A	False (F)
Y	Y	False (F)
A	A	True (T)

(IV)

P1	P2	P1 or P2
A	Y	True (T)
Y	A	True (T)
Y	Y	False (F)

A	**A**	**True (T)**

TASK 2

SUGGESTIONS

- After some basic concepts of Mathematical Logic have been discussed, such as mathematical truth, mathematical implication, mathematical equivalence, this script can be read or given as homework at home to be memorized and acted by the students.

- The teacher can decide to give the information to students on who Eugene Ionesco was through printed handouts e.g:
  Handout or giving the link:
  https://en.wikipedia.org/wiki/Eug%C3%A8ne_Ionesco
  Or can go beyond the task of acting to asking the students to do a quick search on who Ionesco was and present it in their dyads divided for this main task.

- Music: it is always welcome as a background. The teacher can choose it from before or ask for the students to choose an appropriate one.
  Accessories: They can use some mathematical instruments from the classroom.
  Costumes: can be very simple and instructed a priori to be brought from home.

TOOL 32: Prime Numbers in "The Big Bang Theory" by Chuck Lorre And Bill Prady

Prime Numbers and Composite Numbers

55	composite
41	prime
37	prime
49	composite
17	prime

Prime Factorization

a) 15 = 5 x 3

b) 36 = 3 x 2 x 3 x 2 = 3^2 x 2^2

c) 72 = 3 x 3 x 2 x 2 x 2 = 32 x 23

d) 118 = 2 x 59

e) 270 = 5 x 2 x 3 x 3 x 3 = 5 x 2 x 3^3

Try that with the following numbers:

a) $\sqrt{493}$ = 22.2036033112

See if it can be divided by 2, 3, 5, 7, 11, 13, 17, 19

It can only be divided by 17

493 = 17 x 29

b) $\sqrt{2486}$ = 49.8598034493

See if it can be divided by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

I can be divided by 2, 11

2486 = 2 x 11 x 113

c) $\sqrt{11541}$ = 107.429046352

See if it can be divided by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107

It can only be divided by 3

11541 = 3 x 3847

d) $\sqrt{199}$ = 14.1067359797

See if it can be divided by 2, 3, 5, 7, 11, 13

It cannot be divided by any of them, so it is a prime number.

TASK

1. $\sqrt{73}$ = 8.54400374532

See if it can be divided by 2, 3, 5, 7

It cannot be divided by any of these, so it is a prime number itself.

2. $\sqrt{37}$ = 6.0827625303

See if it can be divided by 2, 3, 5

It cannot be divided by any of these, so it is a prime number itself.

3. 73 is the 21st prime number

N = 21 and P_n = 73

21 = 7 x 3

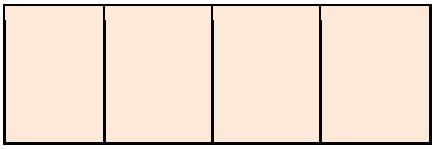
4. 37 is the 12th prime number

N = 21 and P_n = 73

rev(p_{21}) = rev(73) = 37

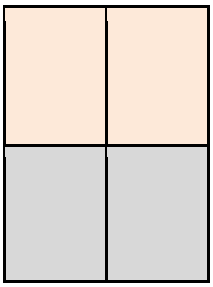
$p_{rev(21)} = p_{12} = 37$

TOOL 33: Prime Number Theory and Partitions in "The Man Who Knew Infinity" by Matthew Brown

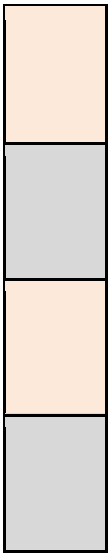


TASK

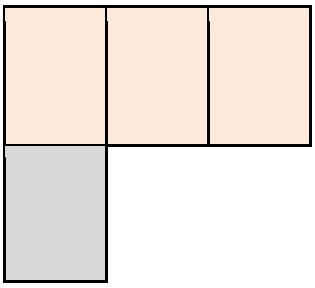
- 4



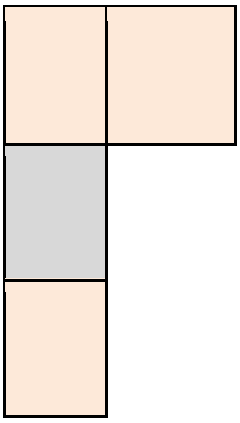
- 2+2;



- 1+3;



- 1+1+2;



- 1+1+1+1;

TOOL 34: Approaching Non-formal Math Through the Movie “X+Y”

The solution of the queries approached within the task is being described here:

Give 0 to face up cards and 1 to face down.

Initially all the cards are face down so the initial row is 1111...

A move can either change 10 to 01 or 11 to 00 and so the resulting number in binary is strictly less than the previous one.

Thus starting from 1111.. the number decreases with each move, hence the moves must eventually terminate at 000…

No matter what face down card you choose you will end up getting all face up cards.

The solution from the movie by the protagonist:

https://www.youtube.com/watch?time_continue=14&v=mYAahN1G8Y8

TOOL 35: Bayes Theorem in "Back To The Future" By Robert Zemeckis

TASK

- P(1955 | F328) and P(1871 | F328) are equal to 0,0.
- We now have two different options: 1985 and 2019

$$P(1985 \mid F328) = \frac{P(1985) * P(F328 \mid 1985)}{P(F328)}$$

$$P(2019 \mid F328) = \frac{P(2019) * P(F328 \mid 2019)}{P(F328)}$$

We need to know what P(F328) is before we continue:

P(F328) = (0,12 * 0,032) + (0,03 * 0,0064) = 0.004

$$P(1985 \mid F328) = \frac{0{,}032 * 0{,}12}{0{,}004} = 0{,}96$$

$$P(2019 \mid F328) = \frac{0{,}0064 * 0{,}03}{0{,}004} = 0{,}04$$

There is a 96% it is in 1985 vs 4% to be in 2019.

TOOL 36: Probabilities in "21" By Robert Luketic

TASK

First, let's see the probability to have each of the possible options:

P(Ace) = $\frac{4}{10}$	P(10) = $\frac{2}{10}$	P(Jack) = $\frac{1}{10}$	P(4)) = $\frac{1}{10}$	P(5) = $\frac{2}{10}$

- We notice that the two cards cannot include a 4 nor a 5, since the sum with the other will not reach 21 points.
- We also notice that we will need the hand to include an Ace and a 10-point card.
- To calculate this, we will need to calculate the probability of having either an Ace and a 10 or an Ace and a Jack.

To write it mathematically, the final calculation will be: P(A∩JUA∩10)

We will need to use the formula P(A∩B) = P(A) x P(B) for each case.

1) P(A∩10) = $\frac{4}{10} \; x \; \frac{2}{10} = \frac{8}{100} = \frac{4}{50}$

2) P(A∩J) = $\frac{4}{10} \; x \; \frac{1}{10} = \frac{4}{100} = \frac{2}{50}$

- Now, we need to know calculate P(1∩2) = $\frac{4}{50} \; x \; \frac{2}{50} = \frac{8}{250}$
- Finally, we will use the formula P(A∪B) = P(A) + P(B) – P(A∩B)

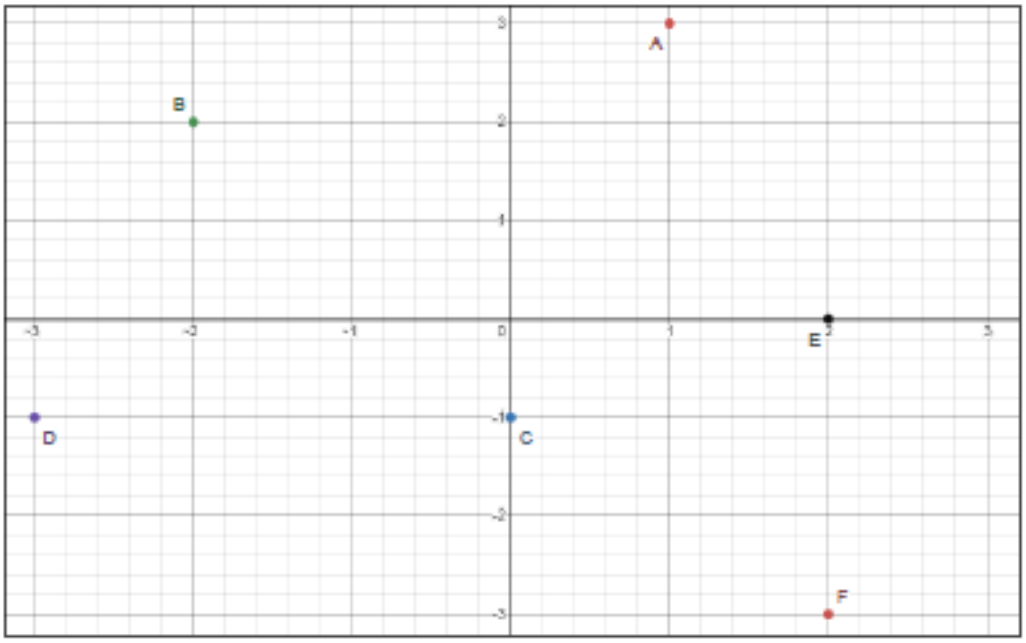
P(1∪2) = $\frac{4+2}{50} - \frac{8}{250} = \frac{30}{250} - \frac{8}{250} = \frac{22}{250} = \frac{11}{125} = 0{,}088$

TOOL 37: Coordinate System Through the Movie "Kingdom of Heaven"

TASK 1

A(2, 0), B(2, 3), C(-3, 4), D(-2, -1) e E(0, -2)

TASK 2



TASK 3

Not applicable.

TOOL 38: Probability and Statistics Through the Movie "Moneyball"

TASK 1

1.1 $RC = \frac{TB*(H+BB)}{PA}$

RC (HOU) = ~738 RC(LAA) = ~680 RC(OAK) = ~767

RC(SEA) = ~677 RC(TEX) = ~666

1.2

HOU = 7,4% LAA = 5,6% OAK = 5,6%

SEA = 0% TEX = 9,6%

TASK 2

2.1

$$SecA = \frac{BB + (TB - H) + (SB + CS)}{AB}$$

SecA(PG) = 0,41 SecA(CD) = 0,21 SecA(JV) = 0,35

SecA(YG) = 0,19 SecA(JM) = 0,2

According to the Secondary Average statistics, Paul Goldschmidt is likely to be overall more effective.

2.2 $RF = \frac{A+PO}{G}$

RF(PG) = 9,06

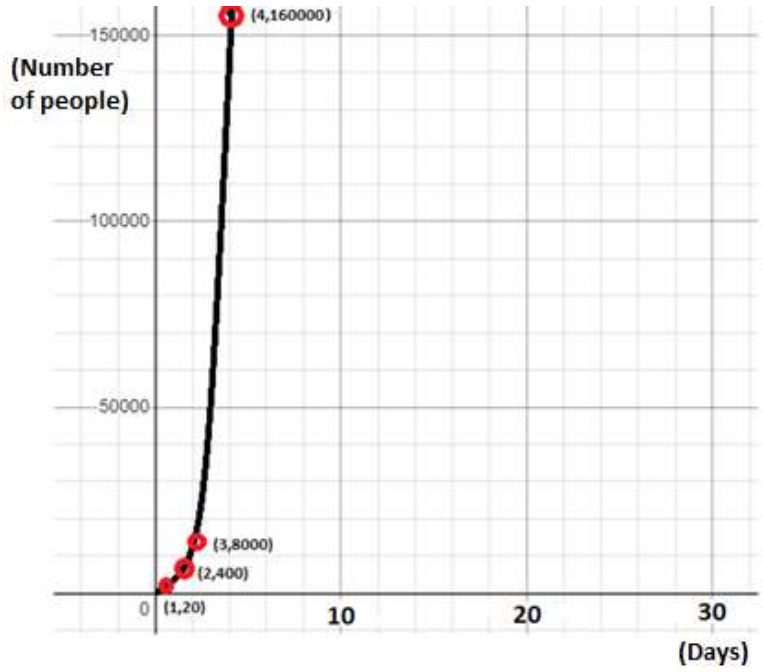
TOOL 39: Exponential Growth through the movie "Pay it Forward"

TASK 1

1.1

Hours	Number of cells	Pattern
1	20 = 20	$y = 20^1$
2	400 = 20(20)	$y = 20^2$
3	8000 = 20(20x20)	$y = 20^3$
4	160000 = 20(20x20x20)	$y = 20^4$

1.2



TASK 2

$$2000(1 + 0{,}15)^6 \ = \ \sim 4626$$

TASK 3

$$81\ (1 \ - \ 0{,}1)^5 \ = \sim 48$$

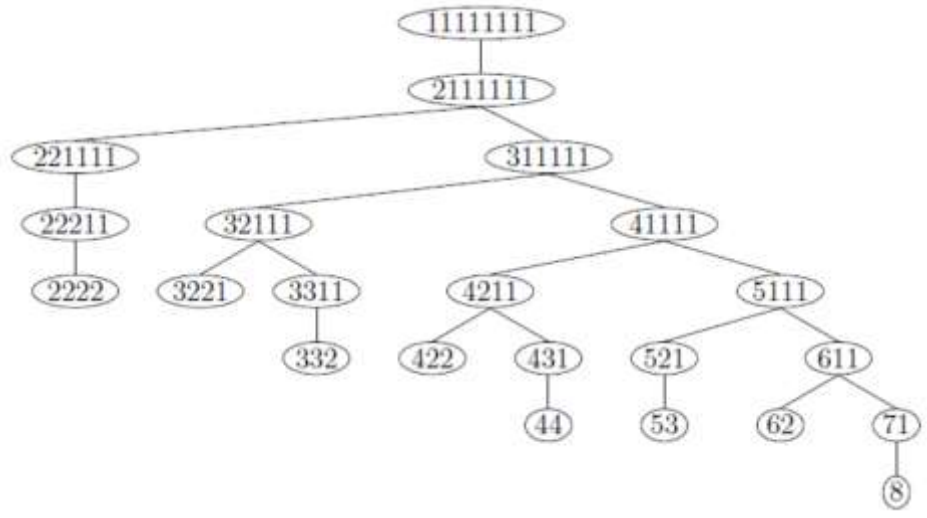
TOOL 40: Approaching Prime Number Theory Through the Movie "The Man Who Knew Infinity"

S= p(4) + p(6) + p(8) = 5+11+22= 38

p(6)= 11

- 1+1+1+1+1+1
- 2+1+1+1+1
- 2+2+2
- 2+2+1+1
- 3+1+1+1
- 3+2+1
- 3+3
- 4+1+1
- 4+2
- 5+1
- 6

Similarly, p(8)=22



TOOL 41: Approaching Derivative of a Function Through the Movie "Hidden Figures"

TASK 1

[No applicable]

TASK 2

$f(x) = x^2$,

So, we must estimate $f(x+\Delta x)$.

Actually, we take $f(x)=x^2$ and instead of x, we use $x+\Delta x$.

So, in the place of x, we put $x+\Delta x$

So, from $f(x) = x^2$, we now get $f(x+\Delta x) = (x+\Delta x)^2$

But we know that if we expand $(x+\Delta x)^2 = x^2 + 2x\,\Delta x + (\Delta x)^2$

⇨ $f(x+\Delta x) = x^2 + 2x\,\Delta x + (\Delta x)^2$

The numerator of the slope formula is:

$f(x+\Delta x) - f(x) = x^2 + 2x\,\Delta x + (\Delta x)^2 - x^2 = 2x\,\Delta x + (\Delta x)^2$

So, the formula gets the following final expression:

$$\frac{\Delta y}{\Delta x} = \frac{2x\Delta x + \Delta x \Delta x}{\Delta x} = \frac{\Delta x\,(2x+\Delta x)}{\Delta x} = 2x + \Delta x$$

Which as Δx heads towards 0:

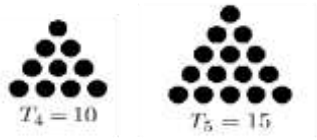
$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = 2x$$

TOOL 42: Approaching Triangular Numbers Through the Book "The Housekeeper and the Professor"

a) $T4 = \sum_{k=0}^{4} k = 1 + 2 + 3 + 4 = \frac{4(4+1)}{2} = 10$

$T5 = \sum_{k=0}^{5} k = 1 + 2 + 3 + 4 + 5 = \frac{5(5+1)}{2} = 15$

b) For T4 the number of dots equals to 10, inasmuch T4 = 10. The length of the T4 triangle equals to n= 4.
For T5 the number of dots equals to 15, inasmuch T5 = 15. The length of the T5 triangle equals to n = 5.

c) 

d) $T_4 + T_5 = 10 + 15 = 25$

e) Two consecutive triangular numbers can be expressed as Tn and Tn-1. Accordingly, the formulas are given below:

$T_n = \sum_{k=0}^{n} k = \frac{n(n+1)}{2} = \frac{n^2+n}{2}$

$T_{n-1} = \sum_{k=0}^{n-1} k = \frac{(n-1)(n-1+1)}{2} = \frac{n^2-n}{2}$

$T_n + T_{n-1} = \frac{n^2+n}{2} + \frac{n^2-n}{2} = \frac{2n^2}{2} = n^2$ which is a square number by definition.

f) The results of (d) do reflect to (e). 25 can be expressed as $5 \times 5 = 5^2$ which is a square number (perfect square).

TOOL 43: Quadratic Function Through the Movie "October Sky"

TASK 1

1.1. 401,5 m.

1.2. Maximum height = 101,5 m; Distance = 200 m.

TASK 2

$$x \in]-\infty;\ 1[\cup]3;\ +\infty[$$

TASK 3

3.1. $h(0) = 1$. The ball is launched from a 1-meter height place.

3.2. The maximum height reached by the ball was 73,2 m and occurred 3,8 s after its launch.

3.3. The ball hit the ground around 7,6 seconds after being launched.

3.4. The ball was less than 30 meters away from the ground in the first 0,9 seconds and after 6,7 seconds.

TOOL 44: Prime Numbers – The Man Who Knew Infinity

[explanation given within the tool]

TOOL 45: Probability in "Mirrored"

TASK

1) [practical task]
2) 1/6
3) 21
4) 1/6, ½, 1/36

TOOL 46: Prime Numbers in "The Curious Incident of the Dog in the Night-Time" by Mark Haddon

Prime Numbers

55	composite
41	prime
37	prime
49	composite
17	prime

TASK

n	$2^n - 1$	$2^{n-1}(2^n - 1)$	n = prime?	$2^n - 1$ prime?	Perfect?
2	3	6	Yes	Yes	Yes
3	7	28	Yes	Yes	Yes
5	31	496	Yes	Yes	Yes
7	127	8128	Yes	Yes	Yes
11	2047	2096128	Yes	No	No
13	8191	33550336	Yes	Yes	Yes
17	131071	8589869056	Yes	Yes	Yes
19	524287	34359476224	Yes	Yes	Yes

TOOL 47: Decrypting with the Fibonacci Sequence in "The da Vinci Code" by Dan Brown

The Fibonacci Sequence - What are the next three numbers?

8+13=21

13+21=34

21+34=55

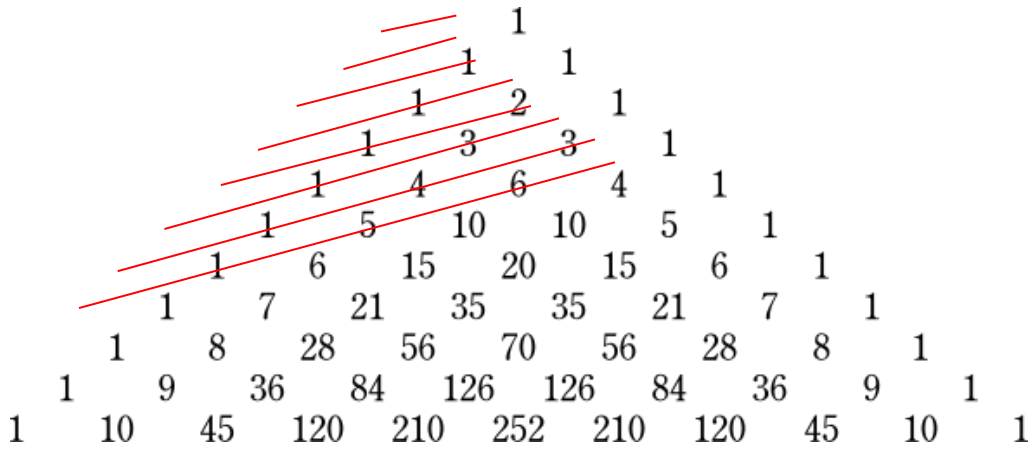
TASK

a) 1st line: The Fibonacci Sequence is mixed

2nd line: Leonardo da Vinci

3rd The Mona Lisa

b) Find the Fibonacci Sequence that is hidden in the figure below, Pascal's Triangle.



TOOL 48: Pilish (π-lish) Writing

TASK 1

3,14159 26535 89793 23846

TASK 2

[Open answer question]

TASK 3

$P = 10\pi\ cm$ (exact value)

$P \approx 31{,}4\ cm$ (approximate value).

TASK 4

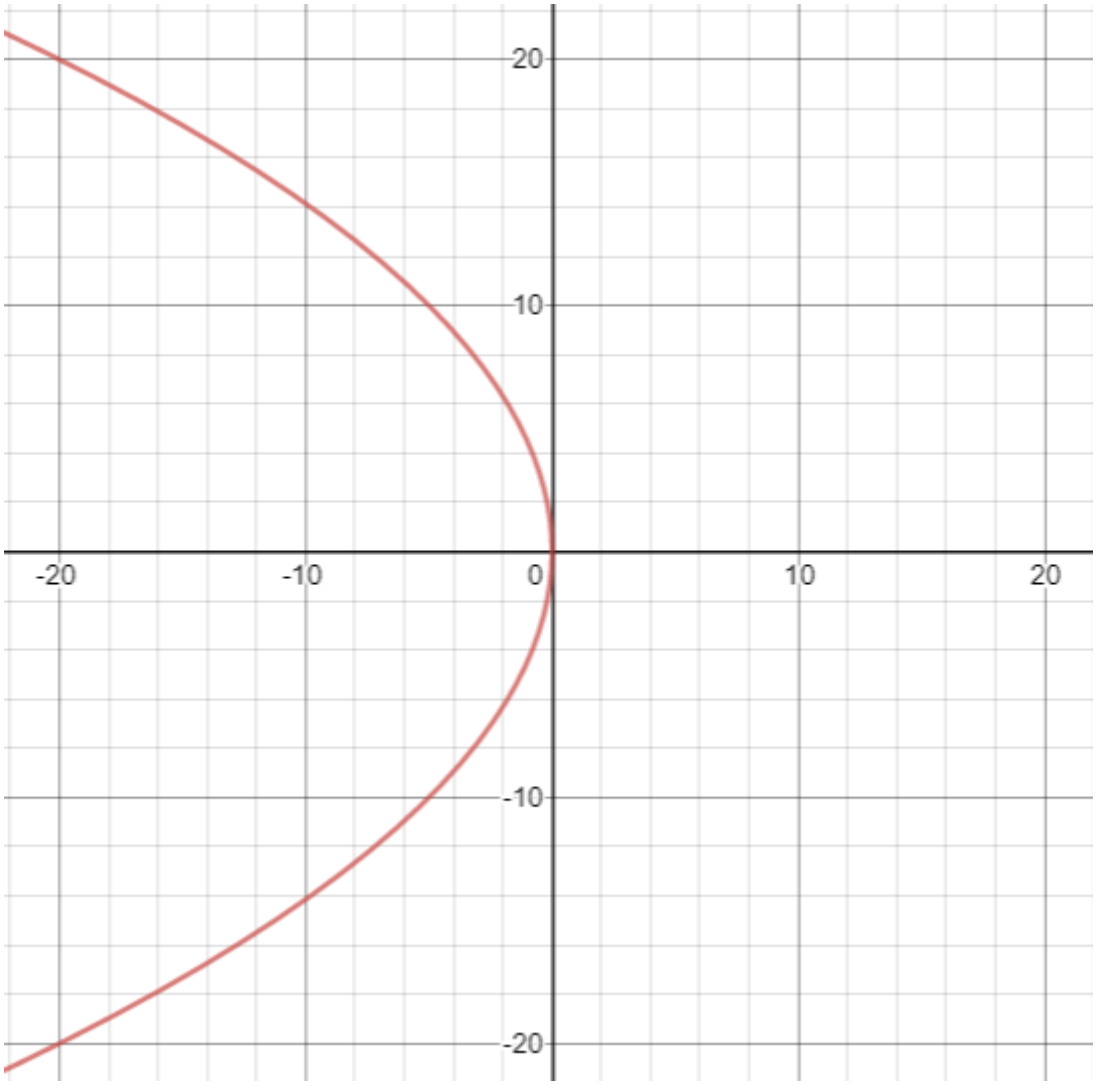
$A = 100\pi\ cm^2$ (exact value)

$A \approx 314\ cm^2$ (approximate value).

TOOL 49: Mathematics in "Alice's Adventures in Wonderland" by Lewis Carroll

Conic Sections

1. The initial equation is y2= 4ax
   Since the focus is at -5 on the x axis, the equation becomes: y2= 4*(-5)x
   The answer is y2= -20x

2. 

3. x2 + y2 – 4x + 8y - 6 = 0

x2 + y2 – 4x + 8y = 6

(x2 – 4x) + (y2 + 8y) = 6

[x2 – 2 *(x) * (2) + 22] + [y2 + 2 * (y) * (4) + 42] -4 - 16 = 6

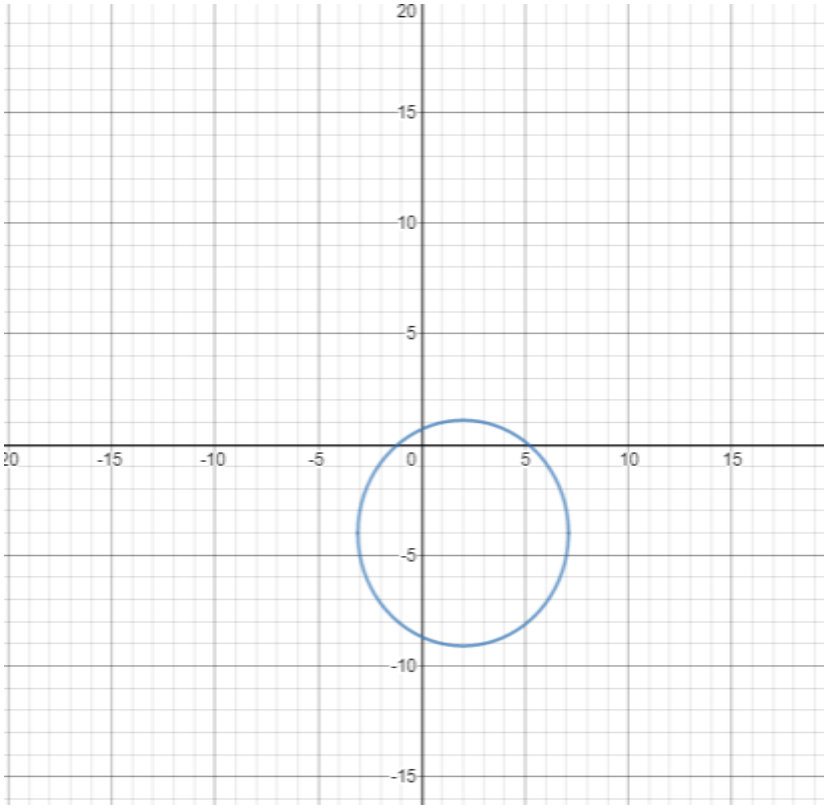
(x - 2)2 + (y + 4)2 = √26^2

Since (y + 4)2 = (y – (-4))2,

We can find the coordinates of the initial equation x^2+ y^2= a^2

The center is thus (2 ; -4), and the radius is √26^2

4.



TASK

1.

2.

a) A hyperbola

b) Possible answer: to show how absurd the new mathematics seemed to him.

TOOL 50: Charts in an Abundance of Katherines

TASK 1

[not applicable]

TASK 2

A) y=- x/2+2

B) y=-x/4+4

C) y=3x+1 and y=-x/3+5

TOOL 51: Moominpappa at Sea and Scaling

[explanation given within the tool]

TOOL 52: Topology in the "Hitchhiker's Guide to the Galaxy"

TASK

1. The correct answer is 2 (the amount of loops (in the number eight)

3. Correct answer is 1, 2, 3, 5 and 7 since there are no loops in those numbers

TOOL 53: Mathematical Poetry

[not applicable]

TOOL 54: Probability in "The Curious Incident of the Dog in the Nigh-time"

TASK 1

The goats and the car

When you start you have ⅓ chance of getting a car. There is ⅔ chance that you get a car if you change your mind and ⅓ chance if you stay with your original choice. The easiest way to show the correct answer is to make a flowchart:

You choose a door					
You choose a door and there is a goat behind it		You choose a door and there is a goat behind it		You choose a door and there is a car behind it	
You do not change	You change	You do not change	You change	You do not change	You change
goat	car	goat	car	car	goat

TASK 2

The colour of cars

a) 1/16
b) 1/32
c) 1/64

TOOL 55: Uncle Petros and Goldbach's Conjecture

Let's try solθτιον:

a) 46 = 23 + 23 = 29 + 17 = 41 + 5 = 43 + 3

b) 38 = 19 + 19 = 31 + 7

c) 14 = 7 + 7 = 11 + 3

d) 22 = 11 + 11 = 17 + 5 = 19 + 3

e) 40 = 23 + 17 = 29 + 11 = 37 + 3

TASK 1:

a) 52 = 23 + 29

YES ☑ NO

b) 76 = 9 + 67

YES NO ☑ Correct Answer:

76 = 3 + 73 = 5 + 71 = 17 + 59 = 23 + 53 = 29 + 47

c) 80 = 59 + 21

YES NO ☑ Correct Answer:

80 = 7 + 73 = 13 + 67 = 19 + 61 = 37 + 43

d) 120 = 73 + 47

YES ☑ NO

e) 64 = 19 + 45

YES NO ☑ Correct Answer:

64 = 3 + 61 = 11 + 53 = 17 + 47 = 23 + 41

f) 92 = 89 + 3

YES ☑ NO

TASK 2:

a) 90 = 31 + 59 = 7 + 83 = 11 + 79 = 17 + 73 = 19 + 71 = 23 + 67 = 29 + 61 = 37 + 53 = 43 + 47

b) 56 = 3 + 53 = 13 + 43 = 19 + 37

c) 88 = 71 + 17 = 29 + 59 = 41 + 47

d) 202 = 11 +191 = 3 + 199 = 23 + 179 = 29 + 173 = 53 + 149 = 71 + 131 = 89 + 113 = 101 + 101

e) 62 = 3 + 59 = 19 + 43 = 31 + 31

f) 94 = 3 + 91 = 11 + 83 = 23 + 71 = 41 + 53

g) 110 = 3 + 107 = 7 + 103 = 13 + 97 = 31 + 79 = 37 + 73 = 43 + 67