Erasmus+

The Art of Maths

The Art of Maths Best-of

Handbook

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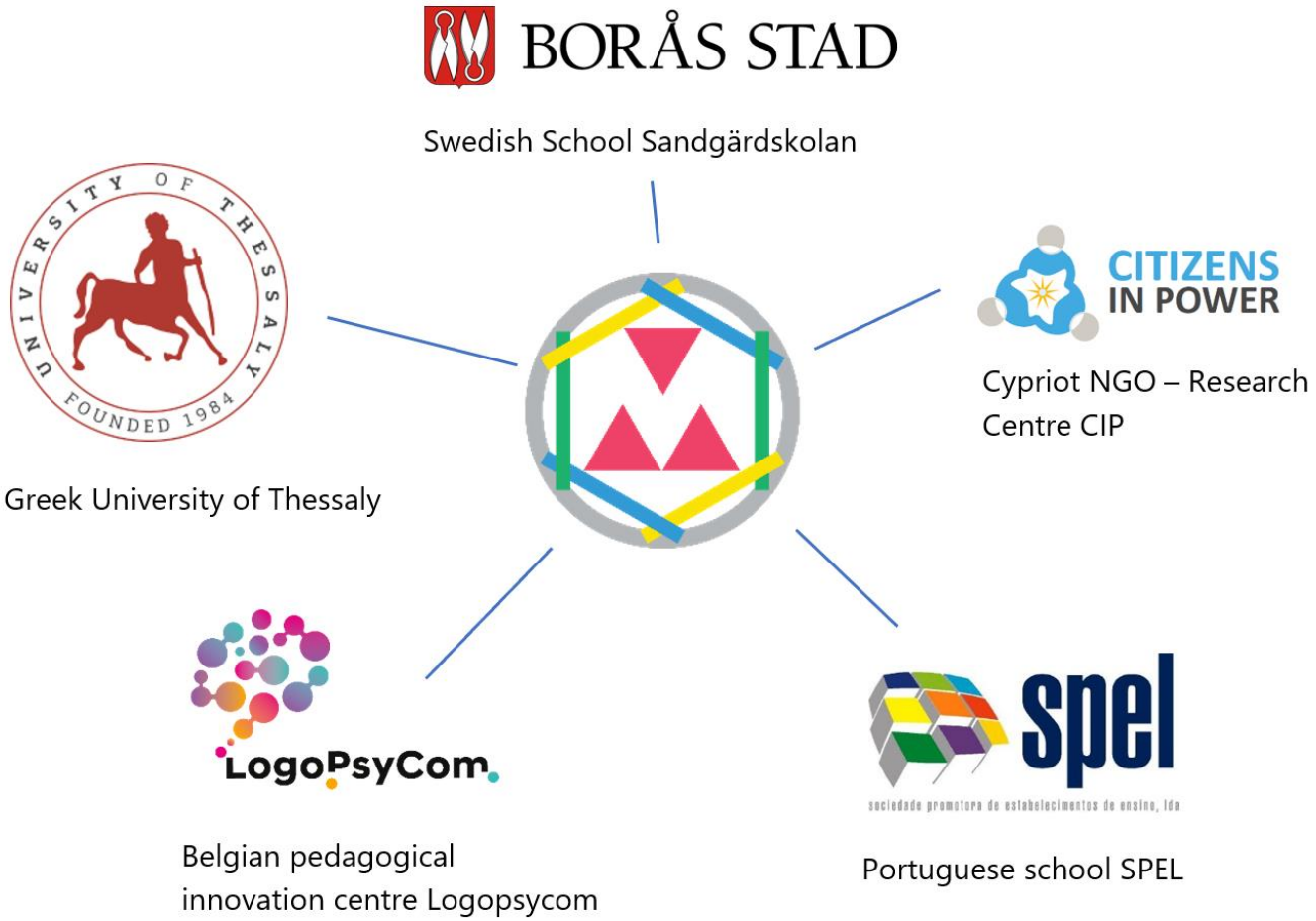
1. Description of the project

The Art of Maths is an Erasmus+ project that aims at **improving the students' achievement and interest in mathematics** in secondary school (age 13-18) by providing tools for teachers **to link maths and artistic creation** to show its creative applications throughout history.

This idea was drawn from the PISA results from 2018 and previous years in math, showing that the **countries in the European Union are lagging behind other countries** such as China, Canada, Singapore and New Zealand, in terms of students' performance in mathematics. Regardless of the students' performance in maths all over Europe, it would be important for them to understand the prominent **role of mathematics in human evolution.** However, the questions that are frequently asked by students, even in high schools, often question the **utility of mathematics in everyday life** and their correlation with other areas of human sciences and culture, showing the need for a **more contextualized learning experience**.

The influence of mathematics in the development of **various forms of arts** throughout history is of great interest, and exactly for that reason has been widely known since ancient times. Particularly, in the recent years, the educational community's research attempts to mitigate the relationship between mathematics and art by using alternative teaching methods, indicating **that the effect of art in mathematical learning is always positive** (Catterall and Waldorf, 1999b; Ingram and Seashore, 2003).

The project "The Art of Maths" is therefore the result of the inspiration drawn by the partnership from the above scientific conclusions. Five organizations are participating in this project:



Each partner has relevant and rich experience in various ramifications of didactics of mathematics through both formal and non-formal methods. The consortium has produced **innovative training modules** with specially designed material that appeals to both maths teachers and students of secondary school.

Therefore, teachers and their students from 13 to 18 years old will have the opportunity to approach the correlation and influence of mathematics in the development of various ramifications of art, including **visual arts, music, cinematography, literature, and theatre**. In addition, they will experience the art development as an alternative method, which could be successfully 'exploited' in maths didactics by simultaneously using ICT-based methods and specially designed innovative tools such as games.

Under this context, the students will make a journey of knowledge in order to investigate **mathematics within the classical art** of Ancient Greece, the proportions of the Parthenon and the notion of the 'golden mean'. Then, they will move to the notions of **'linear perspective' of the Renaissance** and **the 'geometry' of modern art**, through a display of works of art and paintings. They will also discover **'Pythagoreans and the numerical relations of sounds'** in order to proceed to the study of harmonics throughout History. Moreover, they will deal with the mathematical notions, mentality, and history through various parts of **theatrical texts, movies and literature**.

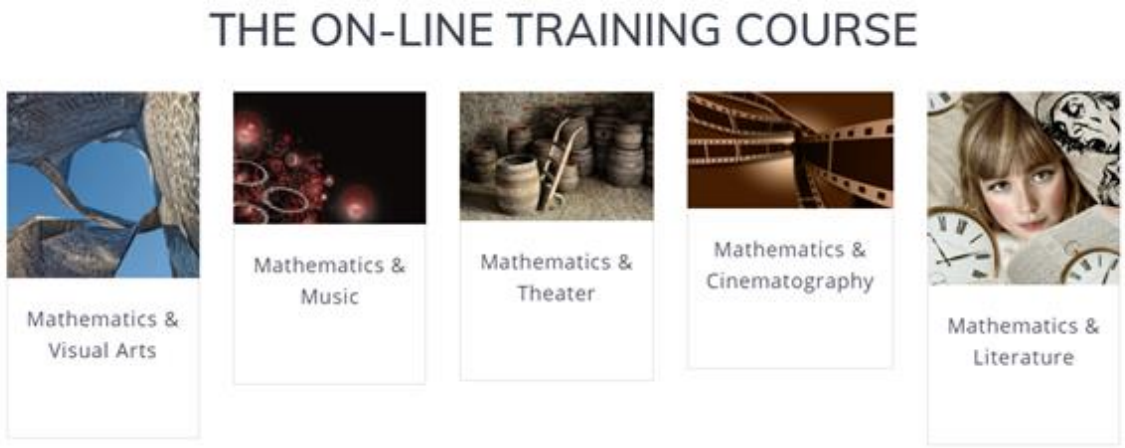
These themes, and many more, are explored through the project tools, which have already been tested by STEM (science, technology, engineering and mathematics) teachers and students in the two secondary schools of the partnership: SPEL and Sandgärdskolan. From this experience, **a selection of the best tools** was made and will be presented in this handbook, along with the feedback from the teachers and students who participated in the Pilot Tests. A unique **E-Library** will provide educators with academic material and articles related with relevant thematic areas explored in the project's tools.

With the material created in this project, students will discover and understand the **applicability of mathematics in other fields**, realizing that it is not a science detached from visual arts, music, cinematography, literature and theatre. The mixture of mathematics and

arts will hopefully create a more pleasant feeling, which would eventually affect the way they accept and approach mathematics in the classroom, **thus cultivating a positive stance towards math**. Another goal of the project is to show teachers and students how to use **digital technologies and innovative tools** in order to absorb knowledge and to fully comprehend a new concept. This will thus also contribute to the modernization of didactic models in teaching maths and STEM, in general. The **on-line training course**, which is available in English, Swedish, French, Portuguese and Greek, is an open resource for all teachers and schools to implement in their daily practice.

2. The sections in "The Art of Maths" tools

The Art of Maths tools can be found in five different languages as Open Educational Resources on our website in the section "online training course". They are divided by thematic area: Visual Arts, Music, Theatre, Cinematography, and Literature, and according to two age ranges: **13 to 15-year-olds** and **16 to 18-year-olds**.



[Online training course of The Art of Maths]

All 55 tools are divided in different sections. The first section, after the cover page, is thus destined to educators and provides the **explanation of the content of the tools** as well as some **practical advice** specific to the tool.

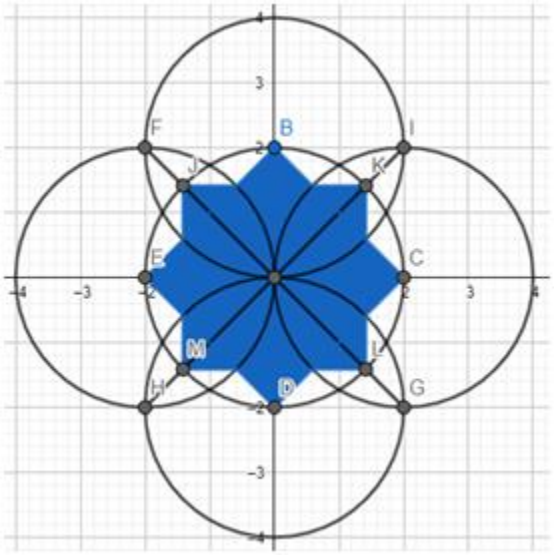
The next section is an **introduction** to the topic of the tool, focusing on the thematic area as well as on the content to be explored in the tool. This introduction can either be used by the teacher alone or be shared with the students to get familiar with the general topic.

The following section is the beginning of the learning path, starting with the **artistic concepts covered in the tool**. The students will get to know the historical context of the artistic movement and its contribution to mathematics. This artistic part will contain videos, images and definitions that intend to engage the students' interest in the contribution of **art in maths and science**. The section ends with a **Glossary**, containing the new vocabulary learnt in the artistic part, thus enriching the students' cultural knowledge, and explaining terms that are necessary to achieve the final tasks of the tools.

[Artworks in Tool 6: The mathematical Art of M.C. Escher]

The next section contains the **mathematical content**. It explains how art can contribute to maths and provide a more thorough exploration of the mathematical concepts covered in the tool. Teachers will be able to use the concepts presented in this section to place the tool in their curriculum and **adapt it to their own methods and program**.

The following section of the tools is **the final task**, linking both the art and the math. It will explore the practical use of what has been explained in the previous sections. Its goal is to provide students with a **practical application of mathematics**, allowing them to use their **creativity**.



[Task result in Tool 2: Islamic Art and Geometry]

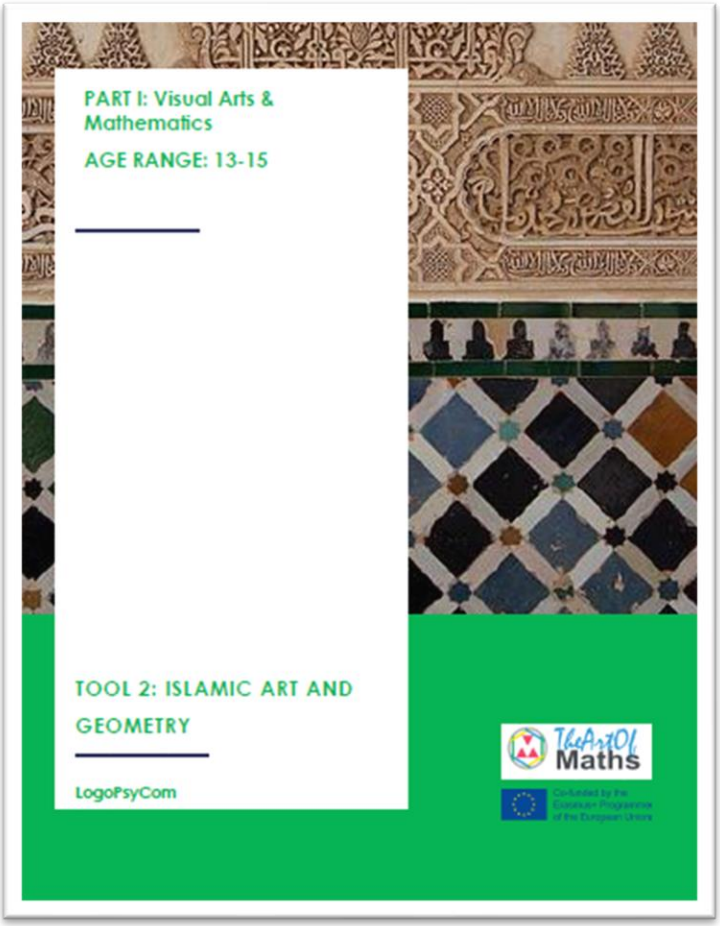
The last section of the tools is **a list of additional resources and information** for both students and teachers to explore after the completion of the activities of the tools. It might help them discover the artistic part a little bit better or give them new ideas on how to do maths with a creative methodology, thus hoping to **further engage and motivate the students**.

3. The five best tools

TOOL 2: Islamic art and Geometry

Thematic Area: Visual Arts

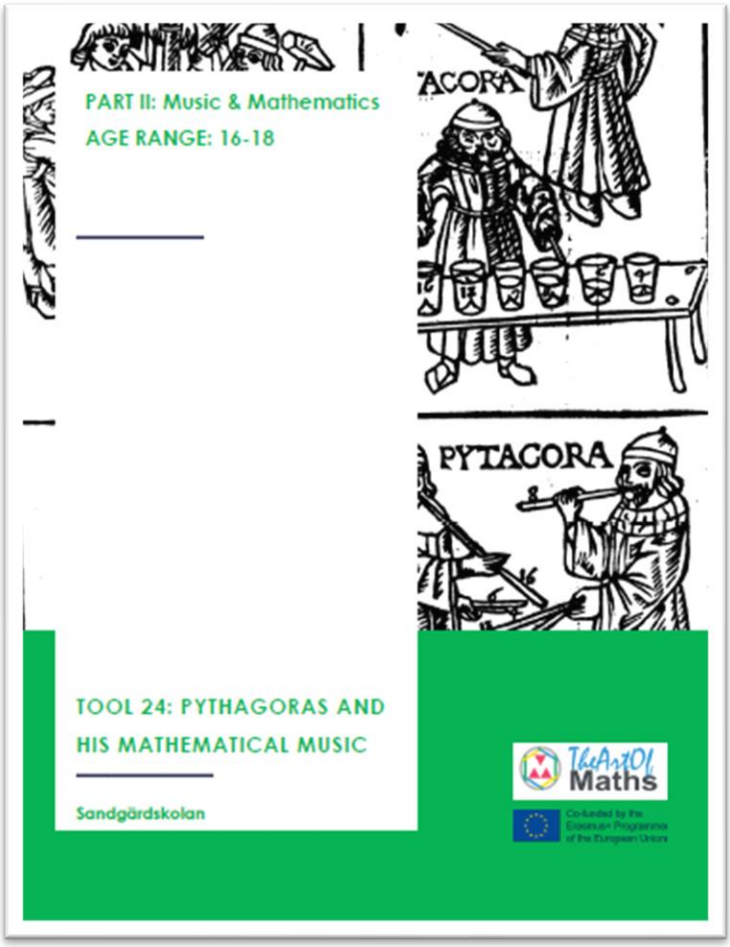
Age Range: 13-15 years old



Tool 2 shows how Islamic Art in the Middle Ages was already using mathematics to draw patterns by geometric shapes in a two-dimensional plane. It thus first presents different examples of Islamic Art, and then links it to the related mathematical concepts, such as the plane, polygons, geometrical transformations and cartesian coordinates. The students' task consists of drawing the most observed star shapes in Islamic Art in a coordinate system.

Link to the tool: https://artofmaths.eu/wp-content/uploads/2020/02/TOOL_2.pdf

TOOL 24: Pythagoras and his mathematical music



Thematic Area: Music

Age Range: 16-18 years old

Tool 24 explores the conception of harmony in Ancient Greece through the discovery of Pythagoras, who was not only a mathematician, but also a great philosopher in 570 – 495 BC. The tool starts with a legend according to which he discovered music theory by listening to the hammers in a blacksmith's shop. The logic behind Pythagoras' theory is then explained in the following section. The final task involves playing music with glass bottles.

Link to the tool: https://artofmaths.eu/wp-content/uploads/2020/01/TOOL_24.pdf

TOOL 30: Volumes in "Seventh Heaven" (The Man Who Counted, Chapter VIII)

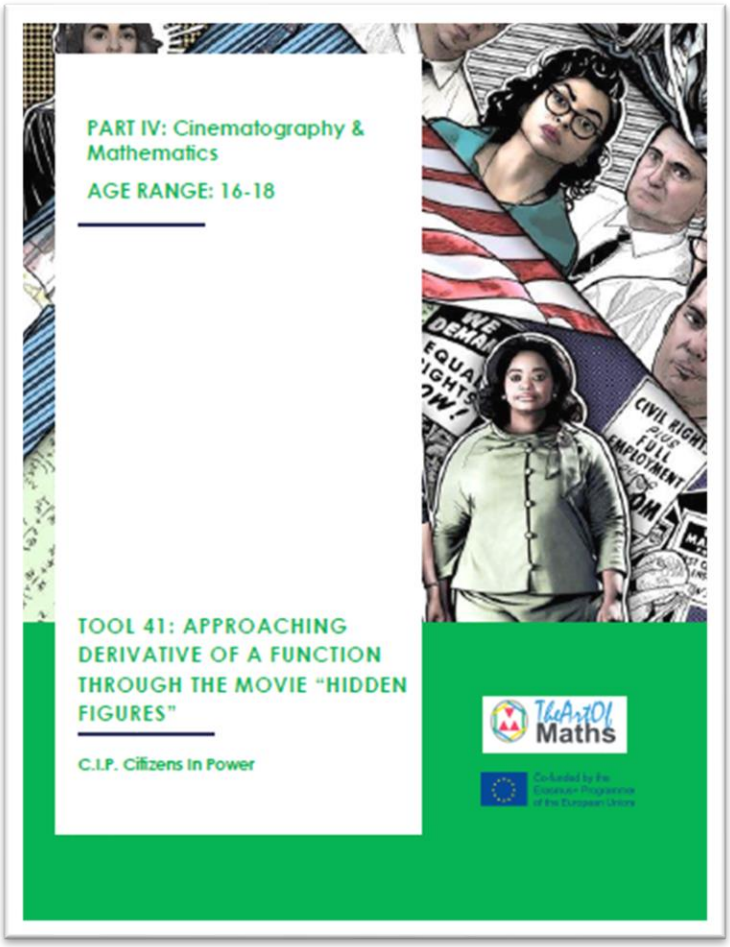
Thematic Area: Theatre

Age Range: 13-15 years old

Tool 30 presents the play "The Man Who Counted" by Malba Tahan, in which a traveller with great mathematical skills solves daily problems in the different places he visits. This tool presents Chapter 8, in which twenty-one wine casks (seven full, seven half-full, and seven empty) need to be shared between three shepherds. This chapter allows to study volumes with the students, which will then be used in the tasks, along with a role-playing activity.

Link to the tool: https://artofmaths.eu/wp-content/uploads/2020/01/TOOL_30.pdf

TOOL 41: Approaching derivative of a function through the movie "Hidden Figures"



Thematic Area: Cinematography

Age Range: 16-18 years old

Tool 41 presents a recent movie called "Hidden Figures", in which three African American women working at NASA greatly contribute to the launch into orbit of astronaut John Glenn. The mathematics of the movie is rather complex, which is why the tool presents the derivative of a function, which is seen at the end of secondary school and is the base on which the protagonists' theories are built. In the task, students will discover more by doing their own research.

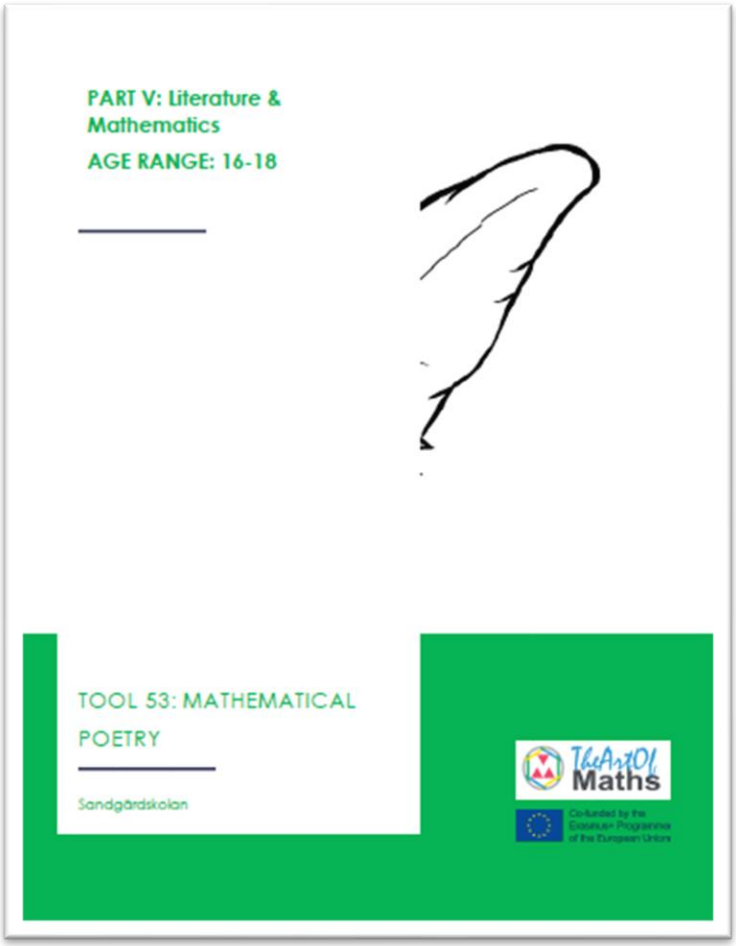
Link to the tool: https://artofmaths.eu/wp-content/uploads/2020/06/TOOL_41.pdf

TOOL 53: Mathematical Poetry

Thematic Area: Literature

Age Range: 16-18 years old

Tool 53 explores the logic behind poetry, through different examples of rhymes and verses. It shows how essential mathematics can be for the rhymes and structure of poetry and emphasizes its use in different types of poems. The tasks then require the students to use mathematics to structure their creativity in their own poems about maths concepts.



Link to the tool: https://artofmaths.eu/wp-content/uploads/2020/06/TOOL_53.pdf

4. Students' and Teachers' feedback

In this chapter, you will discover the feedback collected from the users that were involved with five of our best tools. During our Pilot Sessions in Portugal and Sweden in the autumn of 2019, we had the chance to gather feedback from the participating teachers and students. A **general enthusiasm** was shown among teachers and students regarding this innovative approach to mathematics and they were **highly engaged** in all the proposed activities. The tools were first presented to the teachers of the partner schools SPEL and Sandgärdskolan and a **workshop** was organized with them to prepare the introduction to the material in their courses.

4.1. The Teachers' Feedback

Teachers answered a general questionnaire about all the tools they implemented in class. Their evaluation is therefore more global for all the tested tools.

- **Didactics and pedagogical relevance**

Regarding the didactic aspect of our tools, the majority of the teachers who tested them felt that our tools had clear pedagogical objectives, fostered a positive stance towards mathematics and promoted the **students' self-development**.

- **Content Suitability**

According to the teachers, the selected content was globally meaningful and interesting. They mostly agreed that it is suitable for the set age range and for all learners' needs, including **learners with Specific Learning Disorders**.

- **Language**

Teachers evaluated the language used in the tools as **adapted and suitable** for the learners. The terminology was scientifically correct, and the tools were overall nicely written.

- **Structure**

Since the tools cover **different age ranges** and subjects, enough continuity in the school curriculum was provided. Teachers agreed that the structure and consistency of the tools enables them to ensure some level of **continuity** in their lessons. One of the teachers stated that these tools are "something that many students [14 years old] can **do on their own**, independently"

- **Aesthetics and illustrative material**

Regarding the aesthetics, teachers considered that the tools were appealing to the students and consistent in terms of graphic presentation. They also liked the fact that the format was **well adapted** for learners with specific needs. Moreover, they agreed that the use of images to illustrate the topics covered within the tested tools was appropriate and therefore facilitated the learning and **kept students interested**, especially the ones with difficulties. One of the teachers participating in the tests said that "Many of the pupils continued afterwards and sought more patterns in art [after doing the Geometry in Islamic arts tool].

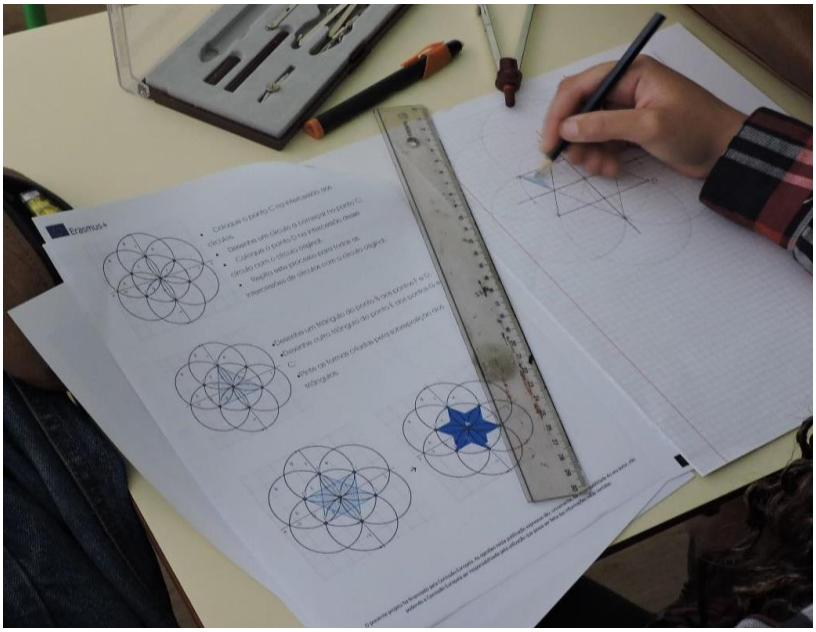
4.2. The Students' Feedback

In order to further improve the material and understand what the students thought about this different approach, they were also asked to fill in a short questionnaire about their experience with the tools. They answered questions such as "What did you like the most about this tool?", "What did you learn with this tool?" and "What do you think could be improved?"

Next follows the students' impressions on some of the best tools chosen for this handbook according to their answers.

Tool 2: Geometry in Islamic Art

25 pupils tested our tool 2 on Islamic Art. They were excited to use their **creativity in a mathematical task** and typically felt more relaxed thanks to this **unusual approach**. For some of them, it brought back memories of drawings and patterns they used to make with their compass. Overall, they had a positive and interesting experience, learning about a whole art form and its mathematical roots. A typical comment from one of the pupils was: "I learned more about the way geometry was used in the Islamic Art".



The comments for improvement were actually more directed towards their maths lessons in general, as they **asked for more explanatory videos and activities alike**.

Tool 24: Pythagoras and his Mathematical Music

30 pupils tested tool 24 about Pythagoras and music. Pupils were interested to find out about Pythagoras' discovery on the relationship between music and mathematics and were pleased with the explanations and the videos selected for this tool. They **very much enjoyed** the task, which consisted of playing music with glass bottles. Many were surprised to learn that depending on how empty/full the bottles were, different sounds could be produced. **Their enthusiasm reached the highest** when they played with different amounts of water in such a manner that the musical scale could be completely produced. Pupils felt motivated to learn by doing something different and about the fact that **mathematics has implication in music composition**.

Similar to the previous tool, their only comments for improvement were that they would like to have this kind of experiences more often in their classroom. This tool showed **a high level of engagement and motivation** for all pupils, even **encouraging the participation of quieter and more inattentive ones**.

Tool 30: Volumes in "The man who counted"

19 pupils tested tool 30 focused on theatre. They discovered the book "The man who counted" and how it can easily be converted to a **role-playing activity**. Everyone was very engaged in the task, which consisted of a role-play. They loved the collaborative aspect of the tool, by **working in groups** and acting together. The tool proved to be **efficient** in the learning of volumes, even though some pupils would have liked a more thorough

mathematical explanation. A typical comment from a pupil was: "I learnt to cooperate and act in front of the class. I also learnt that **many mathematical techniques can be used to solve the same task**".

Tool 41: Derivative of a function in "Hidden Figures"

Tool 41 is about the movie "Hidden Figures" based on the true story of three African American women scientists in charge of the project that first landed man on the moon. In fields in which most successful people are men, it is motivating to see how women can also strive. This fact can possibly **motivate women pupils paving a STEM path/career**, as well, by giving them role models to look up to.

The tool was not tested with students, but the discussions with teachers of the partner schools showed the great potential that the tool could have in terms **of motivation and gender equality**. The students who would use tool 41 would learn how, without Math,

mankind would never have landed on the moon. It shows them a **real-life application of mathematics** and the progress it can bring to our society's evolution.

Tool 53: Mathematical Poetry

Tool 53 is about how writing poetry also requires **following a logical path**. The tool was presented to teachers who expressed their opinion on the tool and reflected on its implementation in the classroom. They were **very engaged**, whether or not Literature was their favorite subject. Teachers from Science, Math, Music subjects, etc. were all looking excited when writing a poem that followed rhythmic and metrical patterns but at the same time was a play on a Maths rule.

They were alluded on how **transversal maths can be** and how it can be adjusted even to a national language school subject. The teachers from the "Portuguese language" subject from that school were surprised that they could take a mathematical tool to their class and give a lecture on the usual topics while still using an **unconventional method to teach math**.

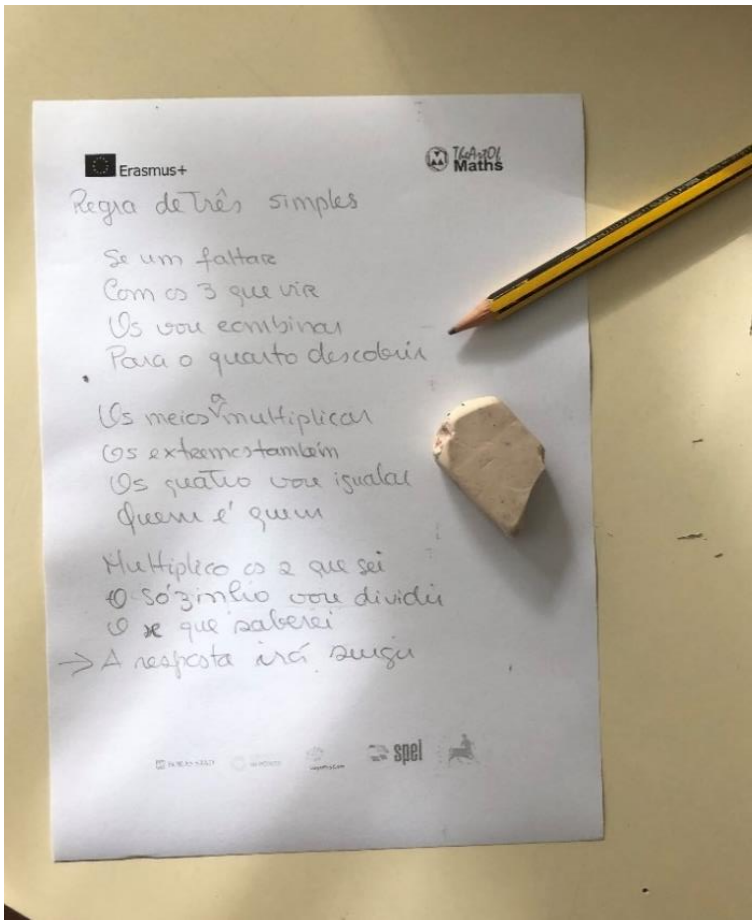
Their enthusiasm was visible, as they were able to put creativity to work. Some poetry that resulted from that task included:

- a poem about the "Simple rule of three"
- a poem about the process behind the simplification of fractions
- a poem on the negative exponent fraction rules

The content was on point and relevant and the feedback on the tool was **clearly positive**.

Poem on the simple rule of 3:

(Loose translation from the original, in Portuguese)



"if one of them is missing

With the three (numbers) that I see

I will combine them

To find out the 4th missing one

Multiply two of them

The opposing ones

I will level the 4 of them

Now, who is who

I will multiply the 2 (numbers) that I know

And divide by the one left alone

I will know the value of "X"

For the answer will show up"

5. Conclusion

The Art of Maths is a project that has enriched both the project partners and the participants in the tests and training sessions. It emphasizes the close link between STEM and different forms of art, and has shown that the combined use of these disciplines has lead to the creation of some of the most famous artworks in history.

In this guide, we have presented one of our favourite tools in each thematic area. Some of these tools were chosen based on the feedback received after the pilot tests in Sweden and Portugal. Others were selected by the partners based on their experiences and discussions with teachers. It was important from the beginning to show that mathematics are not merely a collection of numbers and equations that stand on their own. Maths provides invaluable resources that students can use in their everyday life to develop their creativity and enjoy art from a different perspective. It also seemed necessary to highlight that mathematics are not only used in visual arts with geometrical shapes, but can be at the core of musical, cinematographic and written works of art.

The learners' feedback show the effectiveness of this method in highlighting the importance of mathematics in all forms of creativity. The tools of the project have shown many different applications but are not an exhaustive list of the topics that could be covered in such a project. All project partners strongly encourage teachers to use this methodology and to develop their own STE(A)M courses, thus adding art to their practice. Both students' and teachers' feedback demonstrated that these tools are a great asset in increasing learners' motivation, engagement and interest.

6. Bibliography

- Ingram, Debra; Seashore, Karen. (2003). Arts for Academic Achievement: Summative Evaluation Report. Center for Applied Research and Educational Improvement. Extrait de l'University of Minnesota Digital Conservancy, http://hdl.handle.net/11299/143655 https://conservancy.umn.edu/handle/11299/143655
- Catterall, J., & Waldorf, L. (1999). Chicago Arts Partnerships in Education summary evaluation In E. Fiske (Ed.), Champions of change: The impact of the arts on learning (pp. 47-62). The Arts Education Partnership and The President's Committee on the Arts and the Humanities. Washington, DC. https://www.artsedsearch.org/study/chicago-arts-partnerships-in-education-summary-evaluation/

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